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# DESIGN SPACE VISUALIZATION AND ITS APPLICATION TO A DESIGN BY SHOPPING PARADIGM

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# ABSTRACT

We have developed a data visualization interface that facilitates a design by shopping paradigm, allowing a decision-maker to form a preference by viewing a rich set of good designs and use this preference to choose an optimal design. Design automation has allowed us to implement this paradigm, since a large number of designs can be synthesized in a short period of time. The interface allows users to visualize complex design spaces by using multi-dimensional visualization techniques that include customizable glyph plots, parallel coordinates, linked views, brushing, and histograms. As is common with data mining tools, the user can specify upper and lower bounds on the design space variables, assign variables to glyph axes and parallel coordinate plots, and dynamically brush variables. Additionally, preference shading for visualizing a user's preference structure and algorithms for visualizing the Pareto frontier have been incorporated into the interface to help shape a decision-maker's preference. Use of the interface is demonstrated using a satellite design example by highlighting different preference structures and resulting Pareto frontiers. The capabilities of the design by shopping interface were driven by real industrial customer needs, and the interface was demonstrated at a spacecraft design conducted by a team at Lockheed Martin, consisting of Mars spacecraft design experts.

Keywords: Multi-dimensional Data Visualization, Design by Shopping

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# **1. INTRODUCTION**

Balling [1] proposes a design by shopping paradigm, where a decision-maker first explores a design space and then chooses an optimal solution from a set of possible designs. This paradigm gives more control to decision-makers compared to traditional optimization approaches by allowing them to form their design preferences after visualizing the entire design space, and then choose an optimal design that is based on this preference. The design by shopping paradigm can be classified as an *a posteriori* articulation of preference to solve a multiobjective optimization [2]. This approach is more attractive to decision-makers than an *a priori* approach, because design exploration is followed by a selection process in which the decision-makers have control [1]. Balling states the need for research in two areas to support this process:

- 1. interactive graphical computer tools to assist decisionmakers in the shopping process, and
- 2. efficient methods for obtaining Pareto frontiers.

The focus in our research is to develop a graphical user interface that allows a decision-maker to visualize the design space and implement a design by shopping paradigm. The following functionality was incorporated into the interface:

- 1. Assign variables to glyph and parallel coordinate plots;
- 2. Specify upper and lower bounds of an n-dimensional design space;
- 3. Implement dynamic brushing within glyph, parallel coordinates, and histogram plots to uncover relationships in the dataset (linked views);

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- 4. Visualize different regions of interest, using preference shading and corresponding Pareto frontier identification;
- 5. Select a design from the glyph plot to display its quantitative information; and
- 6. Visualize the glyph plot in stereo.

In the next section, we discuss our research motivation and different systems that have been developed to visualize multidimensional data. In Section 3, we present our interface for multi-dimensional data visualization. In Section 4 we illustrate its use by visualizing different preference structures and resulting Pareto frontiers for a spacecraft design example. Closing remarks and future work are given in Section 5.

# 2. MULTI-DIMENSIONAL DATA VISUALIZATION

It is difficult to display all variables simultaneously in complex, multi-dimensional design spaces due to the limitations of our three-dimensional world. As a result, multidimensional data visualization techniques are needed to view such design spaces in a manner that is understandable to decision-makers. To promote a design by shopping paradigm, an effective graphical user interface becomes very important, allowing an engineer to form an accurate preference of the design problem. The basic idea of data visualization is to graphically display the numerical data, allowing users to gain insight by interacting with the data [3]. With the recent appearance of design problems with very large datasets, many opportunities exist to improve the efficiency, accuracy, and reliability of decision-making in complex design spaces, introducing many challenges in providing effective graphical interfaces [4].

Several freeware and commercially available software packages have been developed to visualize multi-dimensional datasets, and these existing interfaces incorporate many widely used multi-dimensional visualization techniques such as scatter matrices, glyph plots, parallel coordinates, dimensional stacking, reduction of dimensions, linked displays, and brushing. Table 1 summarizes this existing visualization software, along with their corresponding visualization capabilities.

Our design by shopping interface incorporates several visualization techniques from the software listed in Table 1; however, previously developed software visualize data without the capability to "shop" for the best design. The goal of our research is to develop an interface that allows a decision-maker to visualize different preference structures *a posteriori*. Additionally, we would like to visualize tradeoffs between variables by displaying Pareto optimal designs for each preference structure. The interface introduces a new form of optimization by allowing a decision-maker to "shop" by visualizing many preference structures and corresponding Pareto frontiers until an optimal design is selected.

Previous interfaces such as 3DVDM, Glyphmaker, Miner3D, Virtual Data Visualizer, and N-vision have used immersive virtual reality environments. Even though traditional graphic capabilities and interaction devices are powerful tools for assessing computer data, virtual reality provides unique visualization and interaction capabilities not offered by a traditional interface, and these capabilities can enhance a decision-maker's ability to understand complex design problems [5]. Many different interaction devices can be used within an immersive environment, which include voice input, joysticks, forceballs, and gloves [6]. Interface development has focused on the ability to implement our ideas into the immersive virtual reality environment (SeaLAB) at the Applied Research Laboratory.

Active and passive stereoscopic visualization require advanced projectors, graphics cards, converters, and eyewear, and as a result, the mobility of such hardware is dependent on size and weight. In addition to using a four-sided wall virtual reality environment, we have developed software with the capability to display stereoscopic images on projection screens; this mobility allows us to transport the interface to customers and conferences. Using the same code base, the final goal of the design by shopping interface is to support the following display capabilities:

- 1. active stereoscopic display on dual monitor displays using a desktop computer,
- 2. the ability to port the same desktop computer and visualization interface to our four-sided virtual reality environment (SeaLAB), and
- 3. passive stereoscopic visualization using mobile projectors connected to a notebook computer.

Details of the design by shopping interface are described next.

Table 1. Existing Visualization Software

Software	Visualization Techniques
<b>3DVDM</b> [7]	3D glyph plots, Ability to assign glyph attributes to variables, 3D scatter plot matrix, 3D grand tour
<b>CVis</b> [8]	Cloud visualization, Brushing, Linked displays, Child/Parent windows
Glyphmaker [9]	3D glyph plots, Customizable glyphs, Ability to assign glyph attributes to variables, Linked displays, Brushing
Influence Explorer [10]	Histograms, Brushing, Linked displays, Glyphs
Miner3D [11]	3D glyph plots, Ability to assign glyph attributes to variables, Brushing, Stereoscopic visualization
Mondrian [12]	Parallel coordinates/BoxPlots, Brushing, Linked displays, Scatter plots, Bar charts, Histograms, Mosaic plots
<b>N-Vision</b> [13,14]	Isosurfaces, Child/Parent windows, Multiple displays, Glyphs
Partek Pro [15]	3D glyph plots, Brushing
Spotfire DecisionSite [16,17]	3D glyph plots, Ability to assign glyph attributes to variables, Brushing, Parallel coordinates, Trellis plots, Linked displays, Histograms, Bar charts, 2D and 3D scatter plots
Virtual Data	3D glyph plots, Customizable glyphs, Ability
Visualizer [18]	to assign glyph attributes to variables, Brushing
<b>VisDB</b> [19]	Pixel oriented techniques, Glyphs (spiral, axes, and grouping techniques), Parallel coordinates, Stick figure glyphs
Visual Mine [20]	3D glyph plots, Ability to assign glyph attributes to variables, Brushing, External charts display additional dimensions
XdmvTool [21]	Scatter matrix, Star glyphs, Assign glyph attributes to variables, Brushing, Parallel coord., Dimensional stacking, Linked displays
Xgobi/Ggobi [22]	Scatter plots, Scatter matrix, Parallel coordinates, Glyphs, Ability to assign glyph attributes to variables, Grand tour, Projection pursuit, Brushing, Linked displays, Labeling

# 3. DESIGN BY SHOPPING INTERFACE

The software used to develop the design by shopping interface includes The Visualization Toolkit [23], JFC/Swing, and Mathematica, as shown in Fig. 1. The Visualization Toolkit (VTK) is an open-source, object-oriented software that allows a programmer to use Java, C++, Microsoft Visual C++, Tcl, or Python code to access the compiled VTK C++ core classes. The Visualization Toolkit includes Java classes that call VTK core classes, thereby allowing one to use JFC/Swing coupled with VTK for our visualization purposes. JFC/Swing is a platform-independent language, which is used to develop graphical user interfaces that have features such as scroll bars, combo boxes, internal frames, and menus. To perform numerical calculations, a Mathematica kernel is connected to the JFC/Swing interface, using KernelLink included in the Mathematica software. Actions performed in the JFC/Swing front-end call Mathematica functions stored within the kernel to execute and return values back to the JFC/Swing interface.



# Figure 1. Software Integration for Shopping Interface

# Customizable Glyph Plots

Glyphs are graphical icons that display multivariate information using their physical characteristics, such as size, shape, orientation, color, texture, and transparency. According to Kraus [24], interactive exploration of multivariate data with glyph plots requires new graphical interfaces that allow users to generate many displays of the same design space. As shown in Fig. 2, users of the shopping interface can customize the glyph appearance by changing the orientation and connectivity of the glyph arms. The lengths of the arms on an individual glyph are used to display a design's n-dimensional information, which allows for a qualitative analysis in the design space [25]. For quantitative information, a user can click on an individual glyph, and a label appears displaying the n-dimensional information along with its design index.



Figure 2. Customizable Glyphs

# Parallel Coordinate Plots

Parallel coordinates, proposed by Inselberg [26], displays multivariate data by using a polyline that intersects equallyspaced vertical axes. Each polyline in the parallel coordinate display represents one design (see Fig. 3); a polyline's nintersections with the vertical axes of the parallel coordinate plot represent the n variables of a design. The resulting layout of a parallel coordinate plot allows decision-makers to visualize all variables of a design. The orientations of multiple polylines reveal qualitative information between adjacent variables in a parallel coordinate display. For example, diverging and intersecting line segments portray relationships such as positive and negative trends, respectively.



Figure 3. A Design Shown in Parallel Coordinates

#### Histogram Plots

Histograms partition a variable's range in the dataset and summarize the total number of occurrences in each partition. Histograms can be used to visualize the mean, skewness, distribution, outliers, and variance of sampled variables.

#### Brushing

A brush is a user-defined region within a multivariate data set in which designs that fall within this region are highlighted, deleted, or masked [21]. A brush that highlights the data displays the entire multivariate dataset with designs that fall within the brush highlighted from the rest of the dataset. Deleting refers to the removal of all designs that fall within the brush, displaying only the designs that are not within the brush, and masking is the opposite in that only designs that fall within the brush are displayed to the user. Brushing of glyph, parallel coordinate, and histogram plots allows a decision-maker to set limits within the design space and visualize correlations between variables.

#### Linked Views

Linking is the process of displaying information across multiple views of data [27]. Brushing commonly uses linked views, allowing a user to select, delete, or mask a subset of designs in one view; as a result, this corresponding action updates other views of the data by selecting, deleting, or masking the same designs.

#### Preference Shading

The goal in preference shading is to allow users to experiment with different preference weights on variables, observing dynamically how the designs order themselves in response. The user sets a weighting vector by adjusting slider bars and combo boxes within the graphical user interface. The designs are then sorted using a standard weighted-sum:

$$f = \sum_{i=1}^{n} w_{i} x_{i} : WeightingVector$$

$$x_{i} : PerformanceParameters$$
(1)

Equation 1 is used to set the brightness of the glyph, with the brightest glyph as the most preferred design and the faintest glyph as the least preferred design. By introducing slider bars that specify the weighting vector, the designer is able to view regions of the design space that correspond to different relative preferences; Figure 4 shows two such preferences as indicated by the arrow.



(a) Maximum Values Preferred for all Variables





#### Pareto Frontier

Balling also emphasizes the importance of displaying Pareto optimal designs in a design by shopping interface since they offer the best tradeoff between multiple competing variables [1]. A design is Pareto optimal or non-dominated if no feasible point in the design space improves one variable without reducing the value of other variables. The Pareto frontier is a tradeoff curve amongst the different variables in the design space, and the shopping interface allows a user to view Pareto frontiers in the glyph plot. While preference shading uses slider bars to smoothly vary a weight, the only aspect used for determining the Pareto frontier is whether more or less of the attribute is preferred, or if the attribute is to be ignored (i.e., weight equal zero). Using the same preferences that are displayed in Fig. 4, designs that fall on the Pareto frontier are distinguished by dark spheres.

In addition to the visualization of non-dominated designs, the interface can display levels of designs that lie near the Pareto frontier. The interface can visualize Pareto frontier levels by finding all non-dominated designs in the entire design space; once these designs are found, a new analysis is performed with all previously identified non-dominated designs omitted from the analysis. This process is repeated a specified amount of times determined by the user, and all Pareto optimal designs of each successive level of analysis are stored and displayed in the glyph plot. An example of multiple levels of Pareto frontiers is shown in Fig. 5.



Figure 5. Pareto Frontier Levels

Graphical User Interface Layout

Figure 6 displays the front-end of the design by shopping interface. The Visualization Toolkit's Java classes access the core VTK classes that use OpenGL to generate the glyph plot display. Java alone is used to display the parallel coordinates and histogram plots, since these visualizations do not require 3D visualization.

The glyph plot, located in the upper left-hand portion of Fig. 6, displays 7-dimensional information; the three spatial axes represent three variables in the design space, while an additional four variables are visualized by changing the lengths of the four arms on each glyph icon. The Select window adjacent to the upper-right corner of the glyph plot allows a user to specify the number of dimensions visualized in the glyph plot, resulting in the addition or subtraction of dimensions by increasing or decreasing the number of glyph arms displayed by each icon, respectively.

Shown to the right of the glyph plot, the Parallel Coordinates window displays multivariate information of a design by aligning dimensions parallel to each other and using a polyline to display quantitative information of each individual design. The dimensionality of the parallel coordinates plot can also be changed using the Select window adjacent to the upper right corner of the glyph plot. In the Histogram Tool window, histogram plots, displaying variable distributions, are shown below both the glyph and parallel coordinates plots.



Figure 6. Design by Shopping Interface Front-End

The brushing/preference shading controls are located in the Controls Toolkit displayed at the bottom of Fig. 6. A user is able to select which variables are included in the brushing and preference shading analyses by using the combo boxes in the left side of the window. Range sliders specify an interval for brushing operations, which simultaneously update all plots, by displaying designs that lie within the upper and lower limits of the brush. Alongside the brushing range slider bars, regular slider bars and combo boxes change the magnitude and direction of the weighting vector used to calculate the brightness of each glyph icon, displayed using preference shading. The same weighting vector is used to find Pareto optimal designs, which are distinguished from dominated designs in the glyph plot by large, dark spheres.

VTK, along with a stereo compatible graphics card, outputs a signal in frame sequential stereo signal format, and this signal can be used within the SeaLAB environment and on desktop computers to view stereoscopic images using LCD shutter glasses. Converters, such as the xpo.2 by Cyviz [28], can then take the frame sequential stereo signal and generate two separate images, one for the left eye and one for the right eye. These two separate signals are fed to separate projectors, stacked on each other, and a polarized filter in front of the two projectors allows each eye only to view its appropriate image on the preserving polarization screen. Therefore, all stereoscopic visualization requirements can be achieved using VTK. Figure 7 illustrates the usage of LCD shutter glasses to visualize glyph plots in stereo.



Figure 7. Stereoscopic Visualization

# 4. MULTIDIMENSIONAL DATA VISUALIZATION EXAMPLE AND APPLICATION IN INDUSTRY AT A SPACECRAFT DESIGN REVIEW

The data used to illustrate the design by shopping interface was generated using simulation-based design data of a Mars orbiter satellite [29] and was developed by working with subsystem spacecraft design engineers at the Lockheed Martin Space Systems Company – Space & Strategic Missiles division. The capabilities of our interface were driven by the real needs of the aerospace industry customer. The dataset was generated in a related effort for the purpose of researching satellite design methods. A simplified satellite design problem was derived from a representation of the 2001 Mars Odyssey spacecraft, which is currently orbiting Mars (see Fig. 8). The subsystems of power, propulsion, guidance, navigation and control, and structures were selected to demonstrate the multidimensional visualization interface. At the system level, budgets for power, mass, and moment of inertia were used to couple the subsystems.



# Figure 8. 2001 Mars Odyssey Spacecraft Example

Along with design rules derived from standard textbooks on satellite design, the authors used real-world spacecraft design rules and industry expertise from a team of engineers at Lockheed Martin to fully validate the methods. The kinds of spacecraft design rules used were analysis equations for design and sizing of hardware components, spreadsheets and catalogs for sizing and selection, representative component cost data, behavioral models for relating subsystems together, and simplified representative CAD data for spacecraft components. We have since taken the methods described in this paper and started applying them to other aerospace examples based on the needs of their customers.

The primary design variables for this example are:

- $\Delta V$ : The change in velocity available over the mission, highly correlated with the propellant mass
- Dry Mass: the mass of the vehicle without propellant
- *Mass*: The mass of the vehicle with propellant
- *Prop\_Mass*: the propellant mass
- *RI-X*: The rotational moment of inertia of the vehicle about its X axis
- *RI-Z*: The rotational moment of inertia of the vehicle about its Z axis
- *Torque*: The torque of the reaction wheel
- *RW Index & RW Choice*: The index into the catalog of reaction wheels, of which there were twelve choices
- *Payload mass*: The mass of the payload, an independent variable in this exercise
- *Sheet thickness*: The thickness of the core structural cylinder, which varies with the mass of different components to keep the first mode of vibration away from the launch vehicle's excitation frequency
- *Slew angle*: The angle over which the vehicle is slewed by the reaction wheel in an arc about the Z axis
- *Slew time*: The time for the vehicle to traverse the designated slew angle

• *Cost*: The cost of the vehicle

The remaining variables of the design space include Payload Power, Minimum Frequency Mode, and Mission Life. The dataset for this example includes 1500 designs, each design having 17 variables, forming a 17-dimensional satellite design space populated by 1500 unique design configurations.

Three different mappings of the satellite design space are shown in Figs. 9-14, and three different preferences accompany each mapping. To illustrate preference shading, general increasing trends of variables are displayed using black arrows within the initial glyph plot. As a result, different preference structures will highlight different regions of the glyph plots, thereby visualizing the shopping process. Each mapping and set of preference structures is discussed in turn.

- In Fig. 9, the variables shown in the design space are:
- *X-axis*: Rotational moment of inertia about the z-axis
- *Y-axis*: Cost
- Z-axis: Mass
- *Red Arm*: Propellant mass
- *Green Arm*:  $\Delta V$  over the mission
- Blue Arm: Reaction wheel torque
- Purple Arm: Payload mass



# Figure 9. Mapping 1 of the Satellite Design Space

Figure 10 illustrates three different preference structures imposed on the mapping shown in Fig. 9, as indicated to the left of each figure. The resulting glyph plots display different regions of interest that correspond to the general trends displayed in Fig. 9; however, these different preferences also share similar regions with the other two preference structures, thereby visualizing tradeoffs and correlations between variables. For instance, a preference of low cost will lead to a satellite design of low mass and low rotational moments of inertia; however, this preference will lead to designs having limited  $\Delta V$ . Figure 10 also illustrates that different preference structures will lead to different Pareto optimal designs.



Figure 10. Three Preferences Imposed on Mapping 1

The visualization of the design space can be easily changed from a 3D perspective to an orthogonal view, offering additional functionality to the user by reducing a 3D glyph plot to a 2D display. Figures 11-12 plot RW Index, a discrete variable, against slew time using glyph arms to visualize slew angle, torque, and cost. In Fig. 11, the variables shown in the design space are:

- *X-axis*: Reaction wheel index
- *Y-axis* : Slew time
- Z-axis: Dry mass
- Green Arm: Cost
- Blue Arm: Reaction wheel torque
- Purple Arm: Slew Angle

Preferences with high slew angle and high torque highlight designs with large purple and blue glyph arms, respectively. Figure 12 displays vehicles with high torque correspond to the final RW Index selection, and the resulting preference leads to very small slew time values. In addition, each reaction wheel index has a unique range of slew times compared to other reaction wheel indices.



Figure 11. Mapping 2 of the Satellite Design Space



Figure 12. Three Preferences Imposed on Mapping 2

The spatial axes of the glyph plots shown in Figs. 13-14 represent three different mass properties of the satellite, and the three different preference structures are used to visualize how variables affect the mass characteristics of each satellite. In Fig. 13, the variables shown in the design space are:

- *X-axis*: Dry mass
- *Y-axis* : Propellant mass
- Z-axis : Mass
- *Red Arm*:  $\Delta V$  over the mission
- Green Arm: Payload mass
- Blue Arm: Reaction wheel torque



DRY\_Mago



Figure 14 displays a Pareto frontier located in both highlighted and faded regions of the design space. Designs on the Pareto frontier located in faded regions indicate a design's strong preference with respect to one variable but poor performance with respect the other variable in the preference structure. The three preference structures display how cost and  $\Delta V$  influence the mass properties of a satellite. A low cost preference shifts the highlighted regions to the lower portion of the glyph plot, correlating to low values for propellant mass, dry mass, and corresponding overall mass. In contrast, high  $\Delta V$  highlights regions in the glyph plot with high propellant mass and high mass properties of the satellite.

As illustrated in Figs. 15-17, dynamic brushing is displayed over the Prop\_Mass (propellant mass) range, by visualizing three different partitions. Positive correlations between Prop\_Mass and other variables that include  $\Delta V$ , mass, RI-X, RI-Z, and cost exist by looking at the resulting polylines and highlighted regions in the histograms.



Figure 14. Three Preferences Imposed on Mapping 3

Correlations in Figs. 15-17 aid the decision-maker in visualizing tradeoffs within glyph plots of different preference structures. For example, Fig. 10 displays a preference of low mass and high  $\Delta V$ , and a tradeoff curve amongst these two variables is apparent since both variables are positively correlated to each other. Fig. 14 highlights the region whose preference includes high  $\Delta V$ , corresponding to a spatial position of high Prop\_Mass and high mass.



Figure 15. Brush Showing Low Prop\_Mass (Black – Prop\_Mass, Yellow – Correlations on Prop\_Mass)



Figure 16. Brush Showing Medium Prop\_Mass



Figure 17. Brush Showing High Prop\_Mass

# **5. CLOSING REMARKS**

We have developed a graphical user interface that allows decision-makers to view a design space and incorporate a design by shopping paradigm to help identify an optimal solution. Multi-dimensional visualization techniques are used to help view complex design spaces without an *a priori* specification of preference, thereby allowing decision-makers to use an *a posteriori* approach to select an optimal design. Using the design by shopping interface, the decision-maker is able to view regions that correspond to different preferences in the design space, which introduces the capability to "shop" for a desired region or set of good solutions. The design by shopping interface is demonstrated with a satellite design reviews by engineers within a division of Lockheed Martin.

Future work will focus on purchasing hardware to meet stereoscopic visualization needs. To use passive stereoscopic visualization, an active to passive signal converter will be purchased along with projectors, preserving polarization screens, and polarized filters, resulting in a mobile virtual reality setup. Along with purchasing new hardware, the interface will be ported to our immersive virtual reality environment (SeaLAB).

Another focus for future work is to improve the efficiency and availability of the shopping interface. By removing Mathematica from the program architecture, and basing the remainder on JFC/Swing and The Visualization Toolkit, both open-source code, the interface will be freely distributable. Mathematica, which supports rapid prototyping of algorithms to perform underlying matrix and numerical computations, will be replaced by custom Java code. In addition, future work will include demonstrating these new capabilities by continuing to work closely with aerospace engineers from Lockheed Martin to further develop the interface based on real customer needs.

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